Oblique Astigmatism in Camera Lenses— Meridional and Sagittal MTF Response

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When presenting lens performance data in the form of a modulation transfer function (MTF) plot, we usually see separate curves for *meridional* and *sagittal* response. This primarily relates to a lens aberration called *oblique astigmatism*. This article discusses oblique astigmatism and the significance of the terms meridional and sagittal. It begins with a concise review of the concept of the MTF plot.

1 THE MODULATION TRANSFER FUNCTION (MTF)

1.1 Introduction

The modulation transfer function (MTF), sometimes called the Contrast Transfer Function (CTF), is a metric that describes the ability of a lens, under certain conditions, to preserve, in the image on the film or digital sensor, the contrast that constitutes the detail in the scene.

1.2 The test target

Testing of this typically involves the use of reflective test targets, each of which we may think is a pattern of alternating black and white lines with a certain pitch, which is illuminated and at which the camera is trained. And it may in fact be just that.

But, in more precise work, each test target has (if we go along it at right angles to the "lines") a pattern of reflectance that follows a "raised sine" function (my term, and what that means we will see shortly).

In this figure we see a rendering of part of a typical "ideal" raised sine test target:

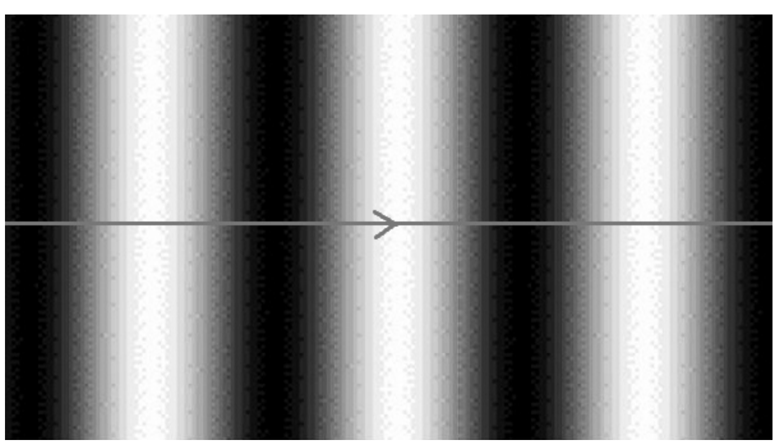

Figure 1. Raised sine test target

If we travel across this test target along a path as shown by the gray line and plot the reflectance, we will get a curve like this:

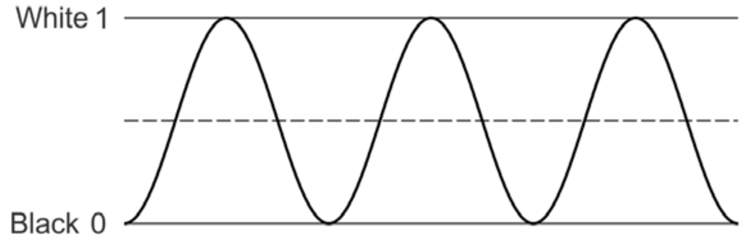

Figure 2. Raised sine test target–reflectance plot

We recognize the familiar "sine function" shape of the curve, but with a difference. The sine function itself is symmetrical about zero, whereas this function is "raised" so its minimum is zero ("black"). (Not surprising, since we can't have "negative" values of reflectance.)

We illuminate this test target with some appropriate uniform lighting. Then, as seen by the camera under test, the pattern of illuminance (L) might be like this:

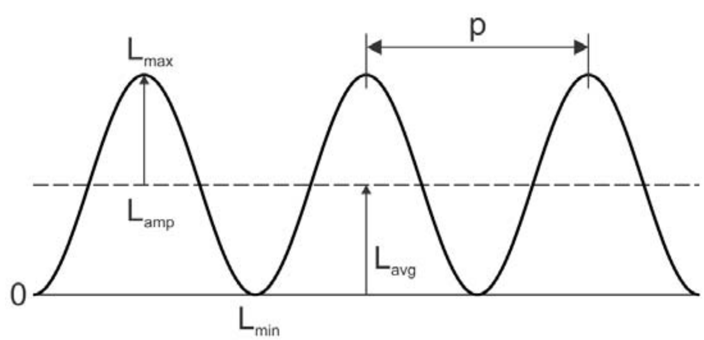

Figure 3. Luminance at the target

Note that the luminance scale (vertical) of this curve is arbitrary, being dependent on the "potency" of the lighting.

This variation of the luminance can be characterized as a "modulation" of the luminance.

The value **p** is the linear pitch of the modulation, the actual distance between one peak and the next on the test target. We can also speak of the "spatial frequency" of the modulation, which is the inverse of the pitch. "Spatial" here reminds us that this will be in terms of the number of cycles per unit **distance**. This is as contrasted to the familiar frequency (as of an AC current), which is formally "temporal" frequency, as it is in terms of the number of cycles per unit **time**.

We might think that the SI unit of spatial frequency would be the *cycle per meter*. But because "cycle" is a dimensionless quantity (a "counting" number), the SI unit of spatial frequency is actually just the *inverse meter* (m^{-1}).

However, in photographic work, the unit used is usually the *line per millimeter*, where "line" (usually) means a cycle of the modulation[1]. The spatial frequency in that unit is the reciprocal of the modulation pitch, **p**, where that is stated in millimeters.

As discussed in the recent footnote, a less ambiguous (and more technically apt) name for that unit is the "cycle per millimeter", but we rarely see that except in scientific papers.

The *depth of modulation*, **m**, for a certain detail location, is conceptually defined as:

$$m = \frac{L_{amp}}{L_{avg}} \qquad (1)$$

where E_{amp} is the amplitude of the sine function of the luminance and E_{avg} is its average value. But that metric is more commonly stated as follows, which is completely equivalent:

$$m = \frac{L_{\max} - L_{\min}}{L_{\max} + L_{\min}} \qquad (2)$$

where $L_{\max}$ and $L_{\min}$ are the maximum and minimum luminance of the modulation for the detail location of interest.

Ideally, when such a test target pattern is imaged by the lens, it would give on the film or digital sensor an image whose illuminance exactly followed the luminance variation of the test target

But, because of various phenomena I will discuss shortly, this is not to be. Inevitably in the image on the film or sensor plane, the depth of modulation of the illuminance will be less than the depth of modulation of the luminance of the test target pattern (but following the same "shape"). We see an example of that in this figure:

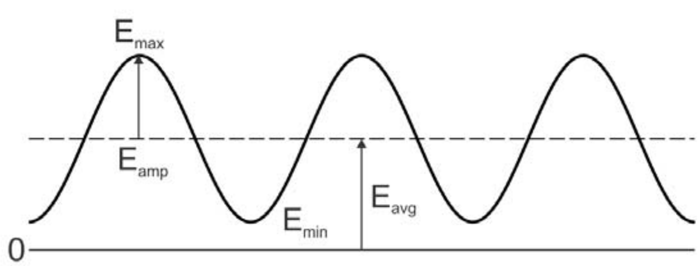

Figure 4. Illuminance at the focal plane

[1] That is based on the view that the test target is a pattern of black lines, seen on a white background. Sadly, often "lines per millimeter" is sometimes used to describe spatial frequency in the sense that the test target comprises alternate black and white "lines", both of which are counted. This obviously gives a spatial frequency value of twice that under the other view. To avoid this ambiguity, sometimes the first outlook is spoken of as being in "line pairs per millimeter". It is best to actually speak of spatial frequency in terms of "cycles per millimeter".

Note that the illuminance (E) scale (vertical) of this curve is arbitrary. Here, the depth of modulation is defined as before, except that the values involved are values of illuminance rather than luminance. We can see that in this example, the depth of modulation, **m**, is less than what it was on the test target (0.75 in fact).

In terms of the "image" created on the film or digital sensor, that might look like this:

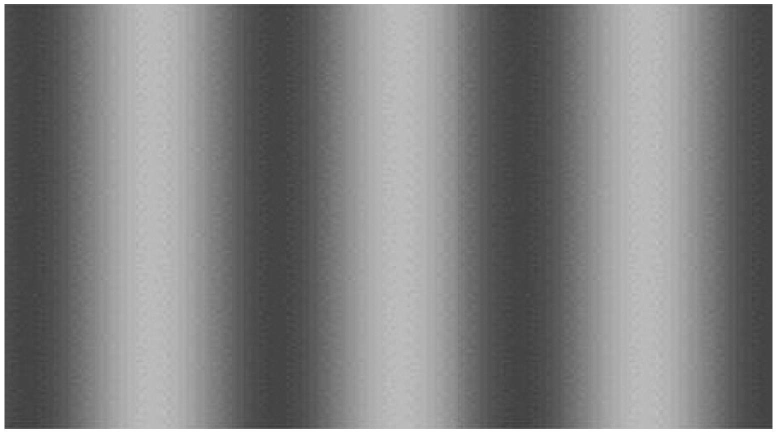

Figure 5. Image at the focal plane

Note that there are phenomena, other than those discussed here, that can lead to **m** at the image being less than **m** at the target. Most notable is the matter of "scattering" of light in the lens, which results in a dilution of the contrast at the image. But I will ignore that here.

1.3 In reality

To make the presentation of the concept of depth of modulation most clear, I assumed a test target with a minimum reflectance of 0 and a maximum reflectance of 1.0. For several reasons, in real MTF testing, a test target is often used where the minimum reflectance is a bit greater than 0 and the maximum reflectance may be substantially less that 1.0 This does not at all disrupt the concept of the depth of modulation on the target and the image, nor their ratio (which I am about to discuss).

1.4 The modulation transfer function itself

The modulation transfer function, MTF, is defined as:

$$MTF = \frac{m_t}{m_i} \qquad (3)$$

where m_t is the depth of modulation at the test target and m_t is the depth of modulation at the test target, at the given target/image point of interest.

That name includes the term "function" because the MTF varies in a consistent way with the values of several variables, notably:

- Model of lens
- Focal length setting (for a zoom lens)
- Focus distance (but usually tested focused at infinity)
- Aperture in use
- The assumed wavelength or spectral distribution of the light involved in the test
- <u>The spatial frequency of the detail (how "fine" the detail is)</u>
- <u>Location in the overall frame (distance from the center)</u>
- <u>The orientation of the detail</u>

For the moment, let us consider a certain lens, set so that all the items not underlined in that list are fixed (or assumed). So we will be concerned only with the three underlined items as the "independent variables" that determine the MTF value.

The curves we speak of as "MTF curves" are plots of the value of the MTF against one of the independent variables, with the other two conveyed by using multiple curves (the "family of curves" approach), each curve corresponding to some choice of values of the other two variables. In effect, those other two variables are treated as *parameters* of the function.

Electrical engineers, familiar with the parallel concept of the response of an amplifier with respect to (temporal) frequency, might expect that we would plot MTF against spatial frequency, with distance from center and orientation of detail as parameters (taken into account by way of multiple curves).

But for some reason, in the photographic lens field, it became the custom to plot MTF against distance from center, with spatial frequency and orientation of detail as parameters. There are only two possible values for orientation of detail (called meridional and sagittal, a major topic of this article), and typically only two (arbitrary) values of spatial frequency (one "low" and one "high") are considered, meaning that only four curves are needed to cover the waterfront.

The following figure is an "illustrative" MTF plot. It is a hypothetical one, from a Nikon tutorial on the matter of MTF plot. It is seemingly for a lens intended for use on a camera using the Nikon "DX" format (28.3 mm diagonal dimension).

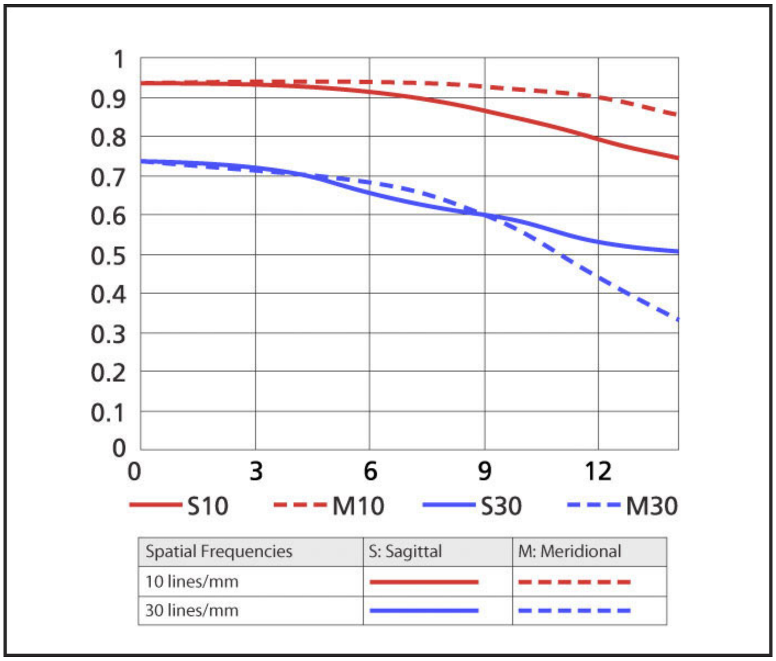

Figure 6. Illustrative MTF plot

The values on the horizontal axis are distances from the optical axis in millimeters. The values on the vertical axis are MTF values. We note that the two spatial frequencies that are plotted are 10 cy/mm and 30 cy/mm (to use the desirable unit), and that the two orientations of the detail, sagittal and meridional, are separately recognized.

Although the concept is straightforward, the terminology used for the two orientations of detail ("sagittal" and "meridional") is rather curious. Explanations of the terminology are often confusing at best and erroneous at worst.

The term *meridional* is often replaced with the term *tangential*, especially in scientific work, and (less frequently) *sagittal* is replaced by *radial (*and in even rarer cases by *equatorial*). I will explain the origin of each term at the appropriate point, but for consistency I will mostly use the terms *meridional* and *sagittal* throughout the main text of this article (since those are the terms usually used in the context that is the theme of this article, lens MTF curves).

1.5 Mechanism of decline in MTF with spatial frequency

Ideally, the lens should produce on the focal plane, from each "point" of the object, a "point image". But in reality, various aberrations lead to the image of a point on the object not being a point image but rather an image figure of finite size (perhaps circular or otherwise elliptical in outline), often spoken of as a "blur figure".

Just as an artist cannot successfully paint fine detail with a fat brush, the lens cannot successfully create an image of an object with fine detail using a light "brush" that is "fat".

One way that this is manifest is that the detail of a certain fineness may be visible in the image but with its depth of modulation (its "contrast", if you will) reduced from what it was in the object itself. And the modulation transfer function, which measures that reduction in "contrast", is one way to quantify this phenomenon, the one of interest to us here.

2 OBLIQUE ASTIGMATISM IN CAMERA LENSES

2.1 Introduction

Astigmatism is a lens aberration in which the lens does not converge all the light coming from a point on the object at a single point behind the lens. Rather, the "cone of light" is converged in one direction (for example, horizontally) at a certain distance from the lens, and in the other direction (in that example, vertically) at a different distance from the lens.

In the human eye, astigmatism results from the eye's lens system (cornea plus lens proper) not being a true "figure of revolution"; that is, not having the same cross section at different planes through the lens axis. This cause of astigmatism is almost absent from camera lenses, although it can result from improper alignment ("decentering") of the individual lens elements. This form of astigmatism affects the focusing of object points whether they are on the lens axis or off.

Oblique astigmatism in a camera lens does not result from any imperfection in manufacture or assembly but rather is an inherent phenomenon of basic theoretical lens behavior. As its name suggests, it only affects object points not lying on the lens axis (and, generally speaking, is more severe the greater the distance of the point from the axis). It can be reduced (corrected) by taking various steps in the design of complex lenses, but it is never practical to completely eliminate it (especially while at the same time acceptably mitigating other types of aberration).

Our concern in this article is wholly with oblique astigmatism and its effect on the MTF of a lens.

2.2 Source and significance

We can understand the source and significance of oblique astigmatism with the aid of Figure 7.

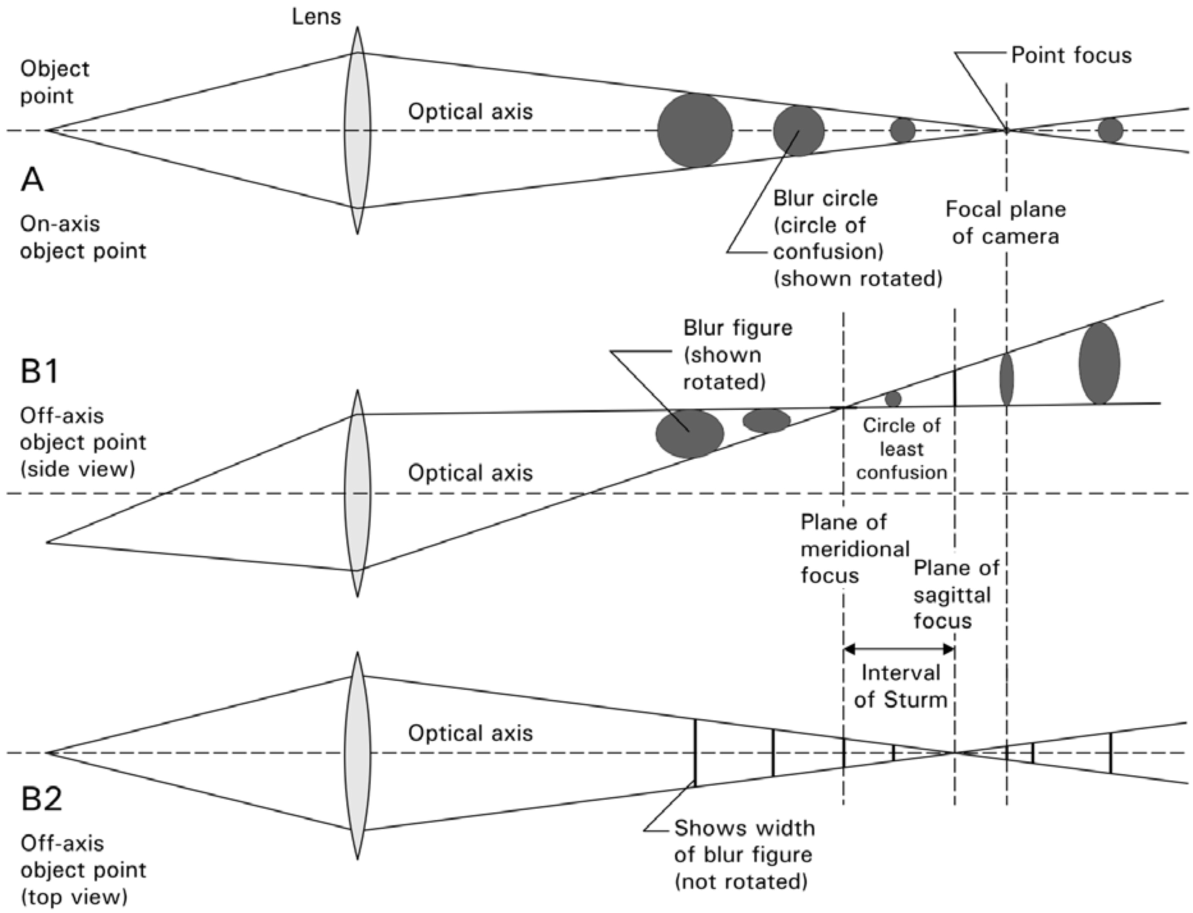

Figure 7. Oblique astigmatism

In the figure, I assume a simple camera lens exhibiting uncorrected oblique astigmatism. Note that I have shown a relatively-unlikely ratio of object distance to image distance merely to make the figure more manageable.

In panel A of the figure, for reference, we see a stylized representation of the ideal operation of the lens in forming a “point image” of a point on the object—in particular for a point on the optical axis. Oblique astigmatism does not impact this situation. And I have ignored spherical aberration, which actually results in all these rays not exactly converging at a single point.

Ideally, the entire cone of light from the object point (through the entirety of the lens) is converged to a point on the *focal plane*, where we will find the film or digital sensor array. (For conciseness, I will from here on speak only of the “film”.) We see that in panel A of the figure. This would look the same in either side or top view.

If we examine the situation in front of the image plane, or behind it, we see that the cone of light has a finite size with a circular cross section. (These cross section portrayals have been rotated into the plane of the paper so we can see them.) We describe that figure as a *blur circle* or *circle of confusion*. If, through incorrect focusing, convergence does not occur precisely at the focal plane, such a blur circle results on the image for each point of the object, resulting in a blurred image overall.

Panels B1 and B2 of the figure show a different situation, one in which the object point of interest lies off the optical axis, in this case at the "6 o'clock" position. The behavior of the lens is not now symmetrical rotationally, so we must look at the field of battle both from above (panel B1) and from the side (panel B2) to see what is going on.

Observing from above (panel B1), we note that the width of the conoid of light decreases to zero at a certain distance behind the lens, a location called the *plane of sagittal focus*. (Don't try and figure out why it is called that—this will ooze to the surface later in our discussion.) Note that this is nearer the lens than the focal plane for the on-axis case.. We might just think that the lens has a shorter focal length for off-axis points, but the situation is more complicated than that.

If we now look at the situation from the side (panel B2), we see that the height of the "cone" of light decreases to zero at a different distance from the lens, a location called the *plane of meridional focus*[2]. (same warning as before—don't try and figure out why it is called that.) Here, it is as if the lens had an even shorter focal length than it exhibited when the action was viewed from above.

Note that here I have greatly exaggerated the distance between the planes of meridional and sagittal focus so we can clearly see what happens. In reality, they are typically very close together. That distance between them is referred to as the "interval of Sturm", recognizing French polymath Jacques Charles Francois Sturm (1803-1855), who did important early work in this area.

Thus there is no place on the emerging conoid of light where all the rays converge to a point—no point at which the film could be placed to receive a proper "point image" of any off-axis point of the object. Thus, no matter where we place the film, the image will be "blurred" in some way.

In view B2 of the figure, we see this conoid from above, with attention to its cross section at the same locations noted in the prior figure. This time, these portrayals of the cross section have not been rotated, so we can only see their widths.

Near the lens (I don't really show this), the cross section is nearly circular. As we proceed farther to the rear, the shape becomes essentially elliptical (with the major axis of the ellipse horizontal), and, at the plane of meridional focus, becomes just a horizontal line (called the *meridional line image*). (I have arbitrarily shown the line with some thickness just so we can see it.) We can think of this as a place where the image of the object point is converged vertically but still spread horizontally. (The formation of the meridional line image is discussed in further detail in Appendix A.)

[2] That being the case, "cone" is not really apt. Formally, the envelope is a *conoid*, and in this case is called the "conoid of Sturm", again recognizing Jacques Charles Francois Sturm. For more on this, see Section 7.

As we continue farther to the rear, the line opens into nearly a circle, and then changes to an ellipse with its axis vertical (not shown). Later, at the plane of sagittal focus, the figure again becomes a vertical line (called the *sagittal line image*). We can think of this as a place where the image of the object point is converged horizontally but still spread vertically.

By the time we reach the focal plane, the cross-section of the conoid has again become essentially an ellipse, this time with its major axis vertical.

In fact, if the camera is still focused as it was in view A, it is this elliptical spot that is the "blur figure" which falls on the film for the off-axis point.

It is this behavior that is described as the aberration of *oblique astigmatism*—the aberration that is commonly referred to as just *astigmatism*.

At a location nearly (but not exactly) halfway between the planes of meridional and sagittal focus the cross section of the conoid takes on the shape of a circle, referred to here as the circle of least confusion (sometimes called the "circle of least diffusion"). It is generally considered that were we to put the film or sensor there, the image would be the "least blurred" overall from the phenomenon of oblique astigmatism.

Of course, the location of the planes of meridional and sagittal focus, and thus also of the circle of least confusion, vary with the distance of the object point involved from the optical axis. So we really can't take advantage of the location of the circle of least confusion to "minimize" the overall blurring of the image.

3 IMPACT ON MTF

A camera lens does not exhibit an ideal MTF plot (MTF = 1 for all situations) if, for any situation, a point on the object is not imaged as a true point on the film, but rather as a "blur figure". Normally, the MTF declines as the spatial frequency (fineness of detail) increases[3]. And the larger the blur figure, the more rapid is the onset of this decline.

Note that the blur figure of which we speak need not result only from misfocus. (If it did, then the lens would exhibit perfect MTF so long as it was focused properly.) The blur figure, even at best focus, results from various lens aberrations (only one of which is oblique astigmatism, by the way).

Suppose the blur figure isn't circular but, as in our example of astigmatism, nearly elliptical (in the example, with the long axis vertical).

If we are talking about the reproduction of detail as we move along a horizontal path across the object (perhaps across a test pattern of vertical lines), the effective

[3] Using again the analogy of the painter, the finer the detail, the more is suffers from the use of a brush of a given "fatness".

diameter of the blur figure is small (the "width" of the ellipse), and the decline in MTF with spatial frequency which that causes has a certain modest degree.

But if we instead consider moving along a vertical path across the object (perhaps across a pattern of horizontal lines), the effective diameter of the blur figure is larger (the "length" of the ellipse), and the decline in MTF with spatial frequency which that causes is greater.

It is for this reason that a difference in MTF, for different directions of traverse across the object, is an indication of the presence of astigmatism (in this case, of oblique astigmatism).

Note that the two directions of traverse across the object are not always vertical and horizontal. That is only true in our example because, for convenience of reference, we chose an off-axis point the was directly below the lens axis.

In the more general outlook, the two directions of traverse of interest (as we might consider at some plane behind the lens) are:

- Along a line passing through the point of interest and the optical axis
- Along a line at right angles to the first line.

It is these two directions of traverse across the image that are spoken of as the *meridional* and *sagittal* directions, respectively. (Yes, at this point this seems backwards. Stay tuned.)

For astigmatism of the type we saw in the figure (the most common type), we can imagine that the MTF in the meriodional direction (vertical in the example) will decline faster with an increase in spatial frequency than the MTF in the sagittal direction (horizontal in the example).

4 THE TERMS

In our work so far, I have used two terms, *meridional* and *sagittal*, to refer to two different directions. I have urged the reader so far not to struggle to understand exactly what those words mean. We are now ready to actually look into their significance.

These terms have synonyms, used in different fields, including:

Meridional = **tangential** = circumferential

Sagittal = radial = equatorial

The first listed term of each group is the one most often used in practical technical information about camera lenses; the ones shown in bold are the most customary in formal technical writing about this topic.

That notwithstanding, we can actually best follow the logic of the terminology by first considering the terms *tangential* and *radial*. They have a direct and obvious meaning from the aspect of geometry, as shown on figure 8.

We see on the figure a circle with a point of interest on its periphery, a radius of the circle through that point, and a tangent to the circle through that point (a line through the point perpendicular to the radius). To avoid the implication that "horizontal" and "vertical" directions are involved here, I have intentionally chosen a point not lying at a cardinal direction from the axis.

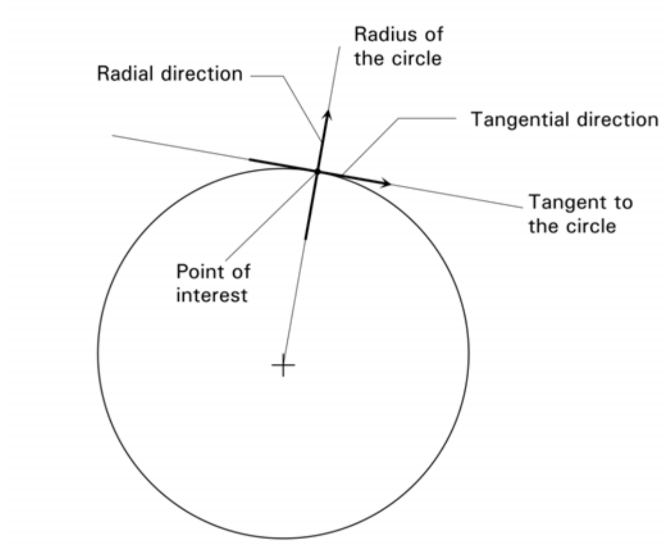

Figure 8. Radial and tangential directions in geometry

The *radial* direction is the direction lying along the radius; the *tangential* direction is the direction lying along the tangent.

As I mentioned earlier, the radial direction is also called the *sagittal* direction. Sagittal comes from the Latin, and means "as the arrow flies"[4]. In this case, the metaphor is an arrow shot from the optical axis toward the point of interest. From here on I will continue to use the term sagittal rather than radial, since it is what we find in most optical and photographic writing.

For those of you who already know the ultimate punch line of this topic, you may think that I have this backwards, considering the way these two terms are used in discussions of MTF and astigmatism. Stand easy—I'm not done yet. Figure 8 merely gives the **geometric** meaning of the terms. Some curious things will happen by the time we get to their application to our subject.

Figure 9 shows how this notation relates to the orientation of the two line images we saw generated in figure 7, views B1 and B2.

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[4] It has the same root as the name of the mythological figure Sagittarius, "The Archer".

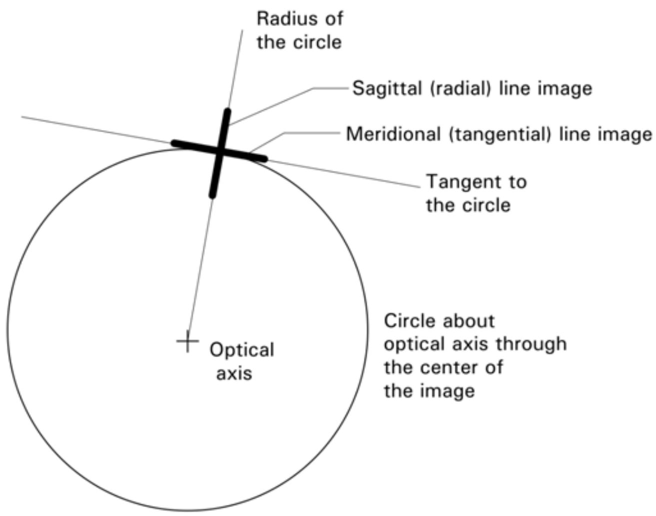

Figure 9. Orientation of line images

It shows the two line images as they would be seen in their respective focal planes. Note that one of them falls in the sagittal (radial) direction, and the other in the meridional (tangential) direction. This is one way the images get their names.

When we test for MTF, we take account of the different behavior of the lens with respect to detail of differing orientation (the result of astigmatism) by testing along "tracks" of differing orientation. Here we see the notation associated with these tracks. [Note that the term "track" is the author's, and is not generally used in technical writing about this topic.)

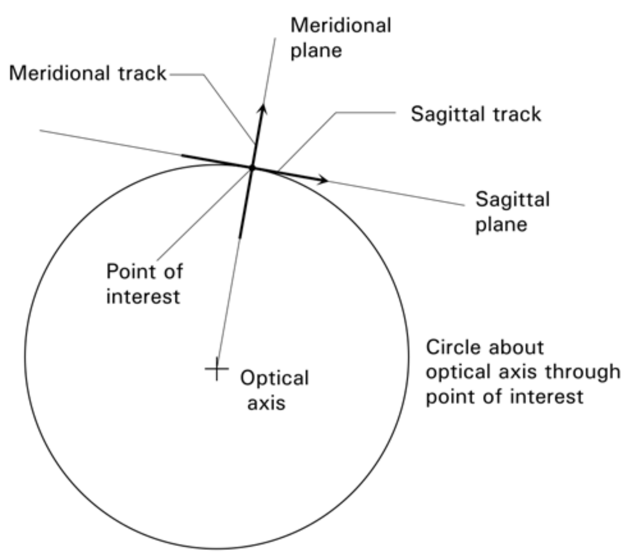

Figure 10. MTF track directions

Note that the synonyms of these names are just the opposite of the geometric names associated with these two directions, one source of confusion in this whole area!

These names are based on the orientation of the two line images shown in figure 7. As we travel along any track, the response of the lens to the detail is based on the narrowness of the line image that is oriented perpendicular to the track. In the case of the track running outward from the optical axis, that is the *meridional line image*. Thus, we speak of the direction of that track as the *meridional* direction. In the case of the track running perpendicular to that track, that is the *sagittal line image*. Thus, we speak of the direction of that track as the *sagittal* direction.

We can see another aspect of this by examining the orientation of the test patterns used to test the response in the two directions—along the two tracks, seen in this figure:

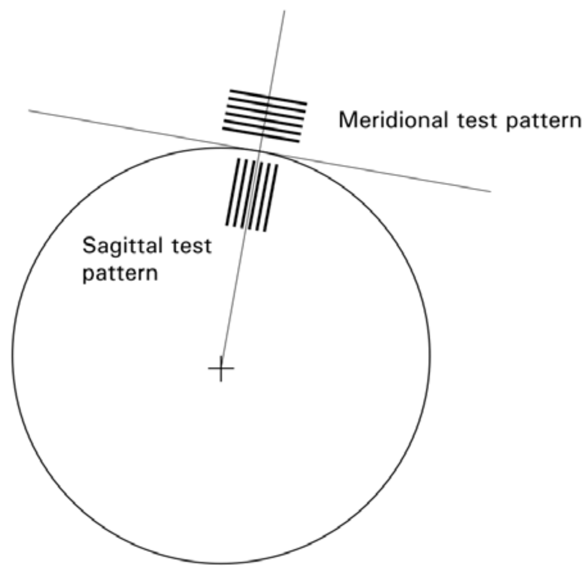

Figure 11. MTF test patterns

For testing along the meridional track, we use a pattern of lines running perpendicular to that track (tangential to the circle). For testing along the sagittal track, we use a pattern of lines running perpendicular to that track (radial as to the circle).

Of course, for actual testing, we have several sets of such lines for each orientation, one for each spatial frequency for which we wish to measure the MTF. And, as discussed earlier, the patterns aren't really "sharp edged" lines, but rather patterns of sinusoidal variation in luminance.

Note that here the orientations of the lines in these two test patterns fit the names sagittal and meridional (or their synonyms) Thus this is perhaps another rationale for the naming of the various directions.

5 THE FANS OF RAYS

Often, in dealing with the matter of astigmatism in lenses, it is convenient to segregate the rays in the conoid emerging from an object point and captured by the lens into two groups having consistent behavior. One way this is done is to identify two different planes in the space traversed by the rays (figure 12).

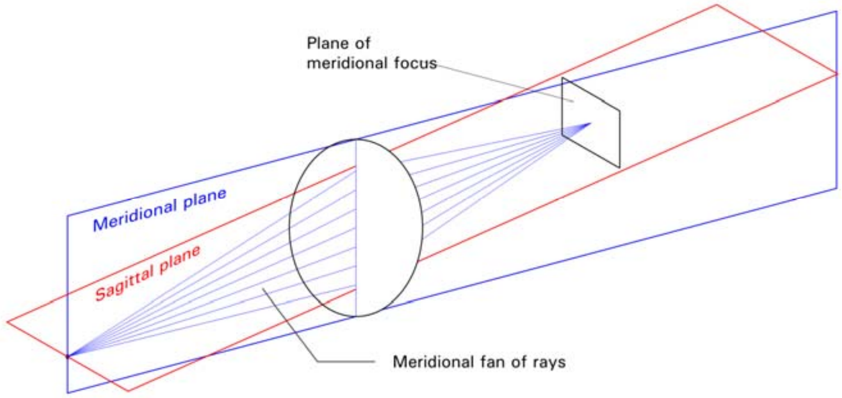

Figure 12. Meridional and sagittal planes; meridional fan

The *meridional plane* is a plane that includes both the optical axis and the off-axis point of interest. In our example, it is a vertical plane, but would not always be, depending on the direction in which the point is off-axis. The *sagittal plane* is a plane that includes the off-axis point of interest and the center of the lens and is perpendicular to the meridional plane. (No, we still can't see yet why they have those names!)

In this figure I also identify a specify subset of the rays in the conoid to which I will refer: the *meridional fan* of rays. These are the rays of the conoid that lie in the meridional plane.

Note that the rays of the meridional fan converge at a point lying in the *plane of meridional focus*. (This point is in fact one point of the *meridional line image* we saw in figure 7, views B1 and B2.)

In this figure I identify the second subset of rays in the conoid to which I will refer: the rays lyning in the sagittal plane, consisting the *sagittal fan*.

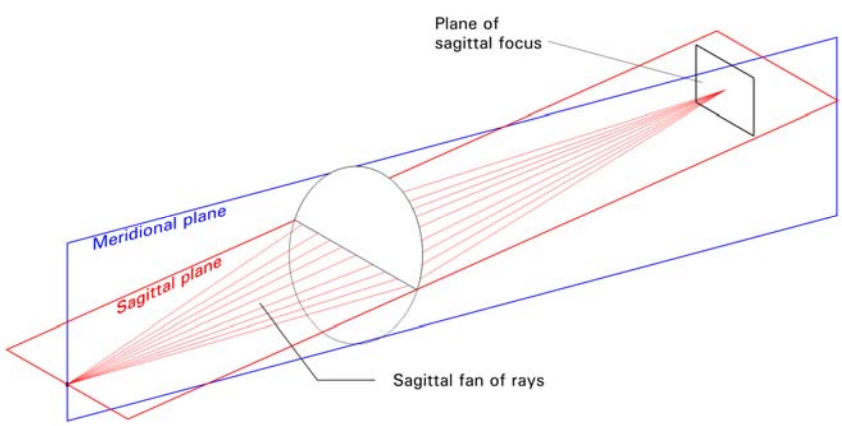

Figure 13. Meridional and sagittal planes; sagittal fan

These are the rays of the conoid that lie in the sagittal plane.

Note that the rays of the sagittal fan converge at a point lying in the *plane of sagittal focus*. (This point is in fact one point of the *sagittal line image*.)

Note that this exercise has not divided into two groups all the rays in the conoid. Rather it has identified two very selective groups of rays (although there are an infinity of rays in each). There are an infinity of other rays in the conoid not in either of these “special” groups.

In these two figures, I have even more exaggerated the difference in the distance from the lens to the meridional and sagittal focus planes (compared to the already exaggerated relationship shown in figure 7) in order to make the difference more obvious.

6 THE NAMES

6.1 Meridional

So far, we have consistently used the term “meridional” for one of the two directions associated with many concepts in this subject, without explaining its basis. In fact, for most lens MTF data published by lens manufacturers, the term *meridional* is used instead of the term *tangential*, used in most scientific work. You’ve already seen the rationale, tortured as it is, for the application of the term tangential. But where did *meridional* come from, and why?

My guess is that the basis of the word is that the plane whose name it bears corresponds conceptually somewhat to a plane through a sphere’s axis, a plane that defines the sphere’s *meridians* (as in the case of the earth).

6.2 Sagittal

What about “sagittal”? Isn’t it also counter-intuitive, for the same reason? Well, it is. If the wonks had stuck with “radial”, there would have been the same seeming inconsistency that there is with “tangential”. But since (trying to show off their knowledge of Latin, I suppose) they had already largely replaced “radial” with “sagittal”, and since nobody could understand what sagittal meant in this context, nothing seemed counter-intuitive (or intuitive either)!

In some fairly rare cases the term “equatorial” is used as an alternate for sagittal, I assume for parallelism to “meridional”.

6.3 The trail of the names

Let’s summarize the trail by which the various items of interest get their “orientation” names.

- The meridional and sagittal line images get their names from their geometric orientations (see figure 9).
- The planes of meridional and sagittal focus get their names from the names of the line images they contain (see figure 7).
- The meridional and sagittal planes also get their names from the fact that the rays in each one are brought to a focus at the correspondingly-named plane of focus, as part of the correspondingly-named line image.

- The meridional and sagittal directions of traverse across the image get their names from the planes in which they lie (see figure 10).

7 THE CONOID OF STURM SITS FOR ITS PORTRAIT

You may well have trouble visualizing the actual shape of the conoid of Sturm. Sadly, I do not at present have a good “3-dimensional” illustration of this. So I have to resort to the use of orthographic projection drawings to hopefully allow the shape to be visualized.

We actually could see it from panels A and B of figure 7, but those are cluttered with many other objects. So here I have isolated the conoid (and the more familiar related structure, a cone):

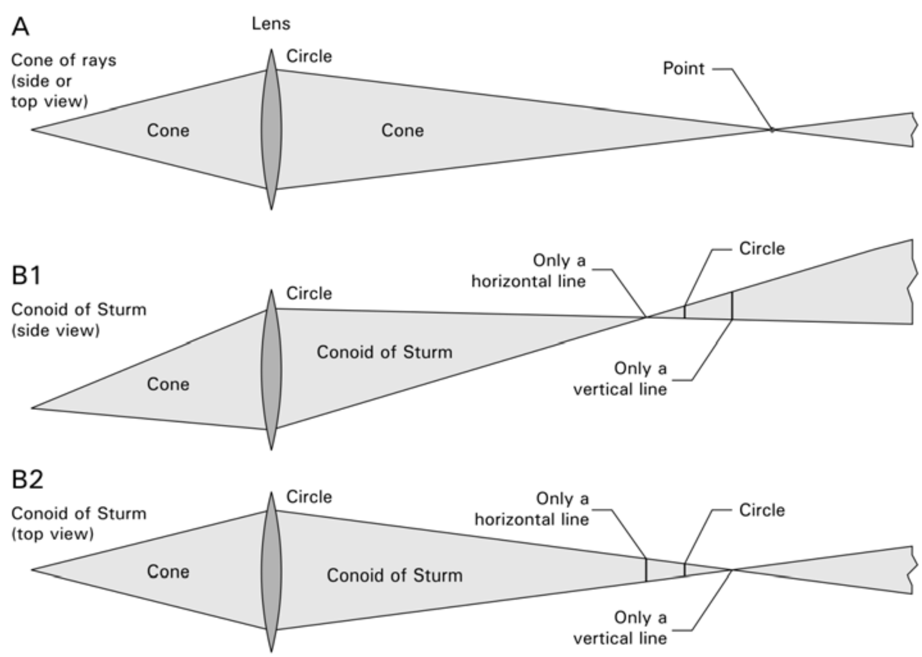

Figure 14. Cone and Conoid

Note that in each case, the set of rays from an object point (on the left) that we consider (since they will pass through the lens), is a cone (not in all cases a right circular cone, but a cone nevertheless).

In panel A, for reference, we those rays as they emerge from the lens to the right forming another cone (which looks the same from any direction). We see that all the rays converge at a point, the apex of the cone (yes, I ignore here the phenomenon of spherical aberration.)

In panels B1 and B2, the rays emerge from the lens, to the right, in the form of a conoid of Sturm, which we see first from the side and then from the top. We see that at one location, all the rays pass through a horizontal line; at a later location, all the rays pass through a vertical line. They never all pass through any point.

We note that as the bounding rays pass from the first line cross section to the second line cross section, they do that in a way that the cross section is always an ellipse (a straight line being the limiting case of an ellipse) starting after the first line with its major axis horizontal, and approaching the second line with its major axis vertical.

At a certain place along the way, the two axes become equal, and the ellipse becomes its special case, a circle.

To help to further grasp the nature of this creature, this image:

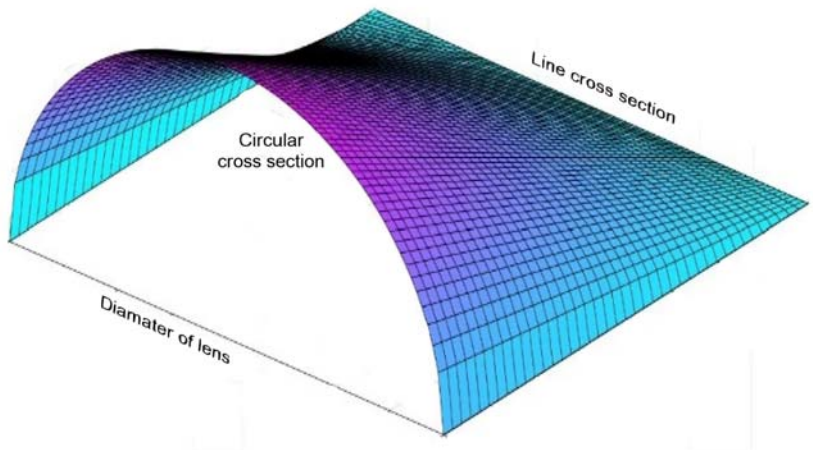

Figure 15. Demi-conoid

Adapted by the author from a figure
in a paper by Joseph Cabeza-Lainez

shows half of a classical conoid, which (in its entirety) corresponds to the part of the conoid of Sturm from the lens to the nearer line focus, and, reversed along its axis, for the part of the conoid lying beyond the farther line focus.

Do not think of the lines on the figure as being rays in our optical situation. They are merely lines used to illustrate the shape of this surface.

8 MISCONCEPTIONS

8.1 Introduction

There are a number of misconceptions floating around that help to confuse us when trying to understand this already-confusing subject. I'll discuss two of the most problematical ones here.

8.2 The "frame diagonal" misconception

We often read (in descriptions pertaining to the MTF testing of camera lenses) that the meridional direction is defined as the direction along the frame diagonal, and the sagittal direction is the direction perpendicular to the frame diagonal.

That turns out to be true in the case where the point of interest happens to lie on the frame diagonal. But that is not the general case, and the true definitions are not based on that presumption. (Note that in the explanations above, the frame diagonal hasn't even been mentioned.)

How did that misconception get started? Here's my guess.

The MTF of a lens varies with several parameters, one of which is the distance from the frame center. In fact, as discussed above, in the form of the MTF chart most often used in presenting lens characteristics, distance from the center of the frame is the independent variable (on the horizontal axis of the plot).

We are interested in the MTF for the full range of distances from the center that we might encounter. Of course, the largest distance from the center occurs at the corners of the frame. Thus we need to be sure to take measurements all the way out to a corner.

Having decided that, we might as well, for the sake of orderliness, take all our measurements—at different distances from the frame center—at points along a diagonal (which of course reaches the corner). That is as good as any path.

Then, for any such test point, the "meridional" direction is indeed along a line from the center of the frame, which of course the diagonal is. And the sagittal direction is perpendicular to that. But this does not constitute the definitions of the two directions.

8.3 The "fan gives a line image" misconception

Even in well-respected textbooks on optical engineering, the statement is often made that "the rays in the meriodional fan form a line image at the plane of meridional focus; the rays in the sagittal fan form a line image at the plane of sagittal focus."

That's just not so. The rays of the meridional fan form a **point image** at the plane of meridional focus; the rays of **the** sagittal fan form a **point image** at the plane of sagittal focus (as we see in figures 12 and 13.)

What does form the meriodional and sagittal line images is **all** the rays emanating from the object point of interest—that is, the entire "conoid" of rays from the object point that is accepted by the lens. We actually see that in figure 7.

In any case, the matter of the formation of the meridional line image is discussed in considerable detail in Appendix A.

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APPENDIX A

Formation of the meridional line image

Even in well-respected textbooks on optical engineering, the statement is often made that "the rays in the meriodional fan form a line image at the plane of meridional focus; the rays in the sagittal fan form a line image at the plane of sagittal focus."

That's just not so. The rays of the meridional fan form a **point image** at the plane of meridional focus; the rays of the sagittal fan form a **point image** at the plane of sagittal focus. The entire ensemble of rays in the conoid is required to form the line image at either location.

This can be most persuasively illustrated by considering the matter of formation of the meridional line image. I will work from figure 16.

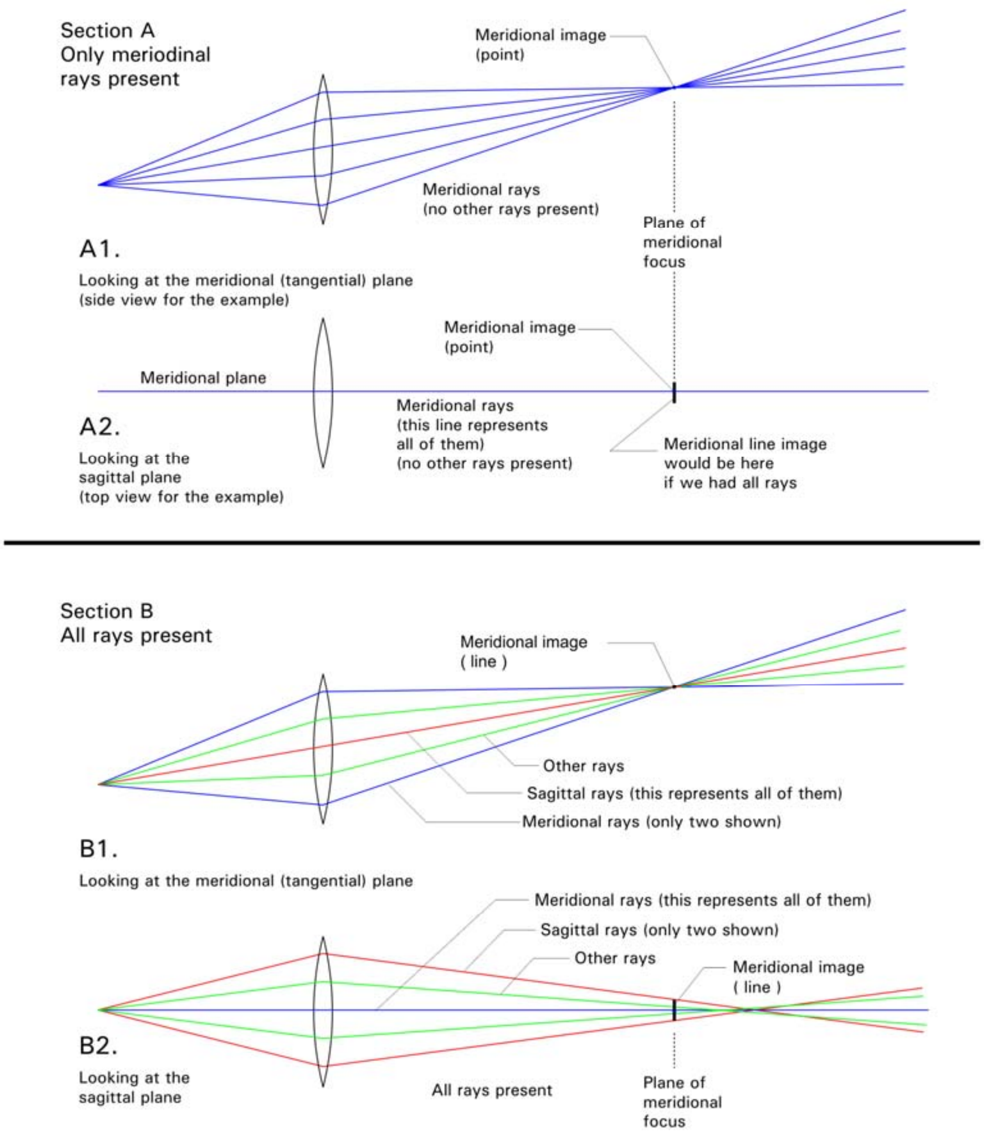

Figure 16. Formation of the meridional line image

Section A of the figure shows the case where there are only meridional rays (the "meridional fan"). In view A1, we see the lens space looking at the meridional plane. We will assume the same situation as in our earlier example—the off-axis point at the 6 o'clock position—so we can conveniently speak of right and left, up and down.

In this view, we see that all the meriodional rays are vertically converged at the plane of meridional focus.[5]

In view A2, we are looking down on the lens space, essentially looking at the sagittal plane (although it is sloping up to our right).

Approaching the lens, the rays of the meridional fan are (by definition) confined to the meriodional plane (which we see edge-on). Passing through the lens, they are not deviated to either side. (An intuitive proof of this relies on symmetry: if they would be deviated, to which side would it be?) Thus the rays remain confined to the meridional plane downstream of the lens.

Thus, at the plane of meridional focus, the image can have no width—there are no rays outside the meridional plane. Accordingly, it is a point image that is formed, not a line image. We show on view A2 where the meridional line image would be, if generated. Note that there are no rays to form any part of it other than its very center—a point image. A line image is not formed by the meridional rays alone.

In section B of the figure, we consider **all** the rays emanating from the object point that pass through the lens. Looking ("from the side") at the meridional plane (in view B1), we see that all the rays are still vertically converged at the plane of meridional focus—that is in fact what that term means.

But looking down (in view B2) we see that the rays are not converged horizontally at the plane of meridional focus—one basic symptom of oblique astigmatism. The overall result in this case is the formation by the ensemble of rays not of a point image but rather a line image—a horizontal one, for the orientation of our example, the one in any event known as the "meridional line image" (as seen in figure 7).

A similar demonstration can be made for the sagittal fan. It is slightly complicated in that we can't rely on simple symmetry to persuade ourselves that the rays remain confined in the sagittal plane after they pass through the lens.

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[5] In fact, if the lens has uncorrected spherical aberration, the vertical convergence will not be perfect. This however does not disrupt the point being made here.