

### **ABSTRACT**

The refractive parameters of eyeglass lenses are specified (in a “prescription”) under a model that considers the refractive effect of the lens as a combination of the effect of both a spherical and a cylindrical lens. In this article we describe this model and its underlying optical principles, and describe how the parameters are usually stated in a prescription. We also look into the fact that a given lens can have its properties stated in two different, but equivalent ways, each used in separate branches of the eye care profession.

We also learn about the “optical cross”, a graphical presentation of the overall refractive behavior of an eyeglass lens.

#### **1 CAVEAT**

I am not an eye care professional, nor do I have any formal training in the practice in that field nor in its own unique branch of optical science. The information in this article is my own interpretation of the results of extensive (mostly quite recent) research into the available literature, through the prism of my own scientific and engineering background and outlook.

#### **2 TERMINOLOGY**

In this article there will be substantial mention of what would be called in most branches of the field of optics *spherical* and *cylindrical* lenses. But in the field of optometry and ophthalmology, it is the custom to speak of these as “sphere” and “cylinder” lenses, and I will follow that convention here.

Similarly, the refractive powers of lenses, about which I will speak extensively, are signed numbers, and the two signs, in most branches of optics, would be spoken of as positive and negative. But in the field of optometry and ophthalmology, it is the custom to speak of these as “plus” and “minus” signs, and I will follow that convention here.

### 3 HUMAN VISION

#### 3.1 Focusing in the human eye

The human eye is organized like a camera, with the retina playing the role of the film or sensor. The lens system is compound, comprising the *cornea* and the *crystalline lens*, each of these being convex lenses. The curvature of the *crystalline lens* can be varied, thus changing its refractive power and thus the net refractive power of the entire lens system. This allows the eye to focus on objects at different distances, an ability spoken of as *accommodation*. For young people with "normal" vision, the range of distances typically extends from infinity to as close as perhaps 10 cm (3.9 inches).

#### 3.2 Deficiencies in accommodation

Typically, with advancing age, the eye's accommodation ability can become compromised (and the same may be true of young people as a result of congenital malformation of the eye or of various ailments). Several types of deficiency are common.

Hyperopia ("far-sightedness") is the deficiency in which the total range of accommodation is "offset out", such that distant objects (even at "infinity"<sup>1</sup>) can be focused on but the near limit is not nearly as close as is normal.

Myopia ("near-sightedness") is "offset in", such that close objects can be focused on but the far limit is not to infinity.

Presbyopia (the term means "old person's seeing") is the deficiency in which the total range of accommodation (the *accommodation amplitude*) is decreased. The remaining limited range may be in the far, intermediate, or near regimes, in the individual case.

The following deficiency may occur in connection with any of the above three, or by itself.

#### 3.3 Astigmatism

Astigmatism (not a deficiency in accommodation) is the deficiency in which the refractive power of the eye's lens system is not the same in different directions. An illustrative result is that if we have astigmatism and look at a cross of thin lines on a card, we can focus so that the vertical line is sharp, or the horizontal line is sharp, but not both at the same time.

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<sup>1</sup> In fact in theory, an eye with hyperopia can focus on objects that are "beyond infinity", but since there are no real objects there, this is of no benefit to the person.

Astigmatism often results from one eye lens system component (usually the cornea) not being rotationally-symmetrical.

### 3.4 Vision correction

Mitigation of these deficiencies is often done with the use of corrective lenses, typically in the form of eyeglasses or contact lenses<sup>2</sup>. Simplistically, their role is to “cancel out” the inappropriate aspect(s) of the eye’s refractive power. Before we discuss this further, I’ll talk a little about lenses.

## 4 LENSES

### 4.1 Focal length

An important parameter of a lens is its focal length. Theoretically, if we have a ray of light from some point on an object located an infinite distance in front of the lens, the lens will bring those rays together at a point a certain distance behind the lens. The distance of that “back focal point” behind the lens is the focal length of the lens.

Actually, in formal optical writing, that distance is called the “effective focal length” of the lens. That makes it sound as if this is not the real focal length, just what the focal length seems to be. But in fact, that is **the** focal length. The odd name was assigned during the emergence of optical theory in order to distinguish this distance from other ways of describing the location of the focal point.

To be precise, the distance to the back focal point is defined as from a certain point “in” the lens, the *2nd principal point*. Despite my use of the word “in”, this point may in fact not lie in the lens at all (like the center of gravity of a donut).

### 4.2 Lens refractive power

The *refractive power* (“power”) of a lens is the degree to which it will converge (or diverge) rays of light emanating from the same point on an object and entering the lens at different points on the front of the lens.

Quantitatively, the power of a lens is the reciprocal of its *focal length* (that is, to be rigorous, its *effective focal length*), ordinarily for the focal length in meters. The formal unit of refractive power is the inverse meter ( $m^{-1}$ ), but the traditional unit, today always used in

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<sup>2</sup> In this article I will only speak of eyeglasses. Most of the principles here also pertain to vision correction with contact lenses, but with a number of specialized considerations, which are beyond the scope of this article..

practical ophthalmology, is the diopter (symbol D) which is the same as the  $\text{m}^{-1}$ . Thus, a lens with a focal length of one meter has a power of 1 diopter.

Sometimes, to make clear that this is the power that is the inverse of the *effective focal length* (we will shortly hear of another power), I will call it the *effective power* (a term not normally used in optical writings).

### 4.3 Vertex power

But in ophthalmology, the refractive power of a lens is described by its (back) *vertex power*. This is the reciprocal of the *back focal length*, which is the distance to the back focal point from the rearmost point on the lens axis (its *back vertex*). In ophthalmology, it is most often spoken of as just the *power* of the lens.

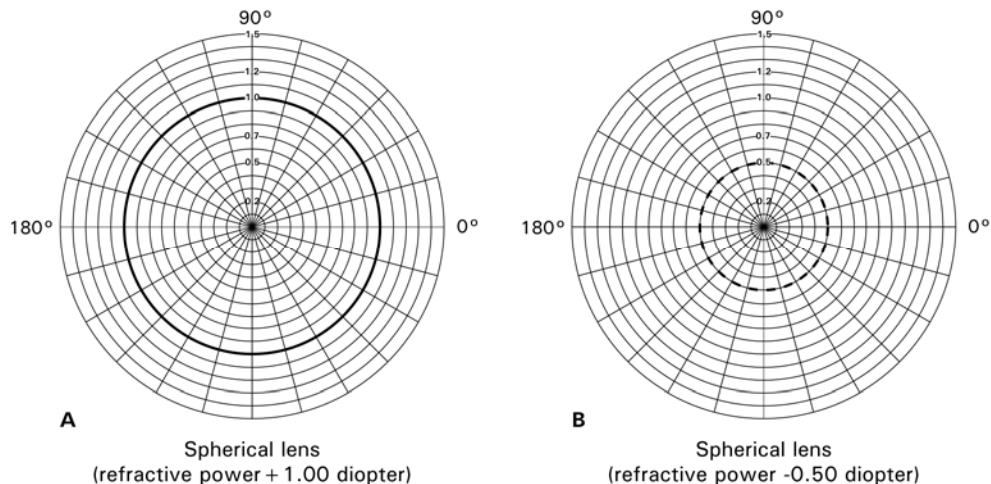
It turns out that the use of this parameter, rather than the (effective) power, simplifies many practical matters in the fields of corrective lenses. Among other things, it allows us to deal with the position of the lens with respect to the eye (which has a great influence on the effect of a corrective lens) in terms of the visually-obvious back vertex of the lens, rather than in terms of the second principal point of the lens (whose location we usually can't immediately recognize, and which may not even be within the lens itself).

### 4.4 Spherical lenses

In ophthalmic work, a *spherical lens* is any lens that has rotational symmetry, whether or not its surface is actually a portion of a sphere. A spherical lens exhibits the same power along any direction.

A converging lens (which has a positive focal length) has a positive power. A diverging lens (which has a negative focal length) has a negative power. But in ophthalmic optics, "plus" is used rather than "positive", and "minus" instead of negative. And I will follow this convention from here on.

We can present the variation (if any) in the refractive power of a lens with direction on a polar chart. In figure 1, panel A, we see a plot of a spherical lens with refractive power +1.0 D (a converging lens). This is a trivial case, and hardly requires a chart to explain. But we show the plot here to establish the format and notation.



**Figure 1. Spherical lens—power plot**

The radius to the curve in a certain direction indicates the refractive power (in diopters) for that direction. Recall that a “direction” here means both ways: either way along the line at a certain angle (called a *meridian* in optometric practice). Because of that symmetry, we only need to plot half the curve. But I show the curve for a full  $360^\circ$  for aesthetic completeness.

The usual scientific convention for angle is followed, with the angle reference ( $0^\circ$ ) being to the right (but there is a wrinkle, about which more shortly) and the angle measured counterclockwise from that.

It is difficult to express minus values on a chart in polar coordinates—a “minus” radius would put the point on the opposite side of the chart, where it would just look like the (plus) value for an angle  $180^\circ$  from the actual angle.

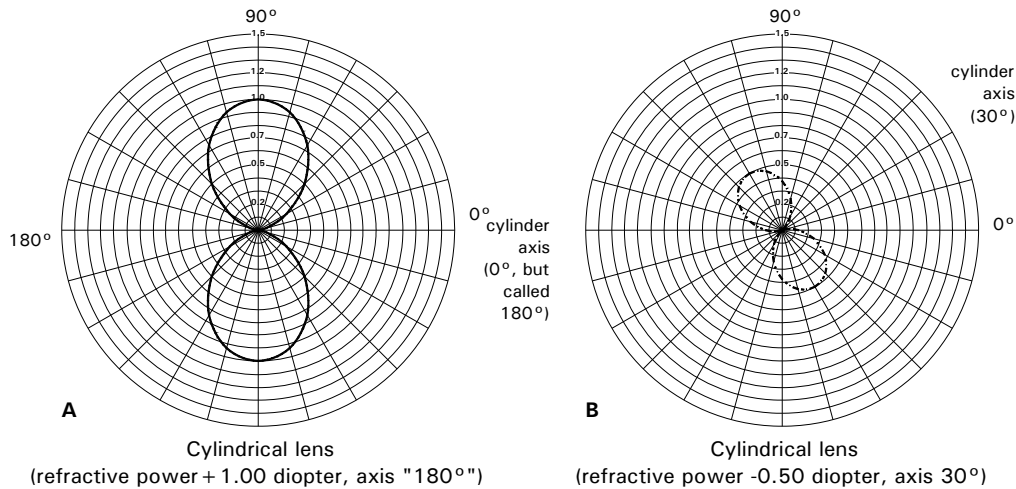
To escape this difficulty, here I will plot minus values of the refractive power as a dotted line. And we see that in figure 1, panel B, the plot for a spherical lens with a refractive power of  $-0.5$  D (a diverging lens).

#### 4.5 Cylindrical lenses

A cylindrical lens has a surface that is a portion of a cylinder (which may or may not be exactly a circular cylinder). A cylindrical lens exhibits a certain power (its “rated” power) in one direction

(perpendicular to its axis). Along its axis, it exhibits zero power. At intermediate angles, it exhibits intermediate values of power.<sup>3</sup>

We see this illustrated in figure 2 for two cylindrical lenses, one with a plus power and one with a minus power.



**Figure 2. Cylindrical lens—power plot**

The power varies as the square of the sine of the angle of the direction of interest from the axis direction of the lens.

#### 4.6 Composite lenses

Imagine that we combine a spherical lens and a cylindrical lens (and we assume here the convenience of the fanciful “thin lens” conceit, which, although impossible to have in practice, makes all the math work out in a very simple way).

In the direction of the cylinder lens axis, where the cylinder lens has zero power, there is no effect of the cylindrical lens on the overall result. In the direction at right angles to that, the power of the cylindrical lens fully combines with that of the spherical lens (taking into account the applicable algebraic signs) and so we have a power different from that of the spherical lens alone (perhaps even of the opposite sign).

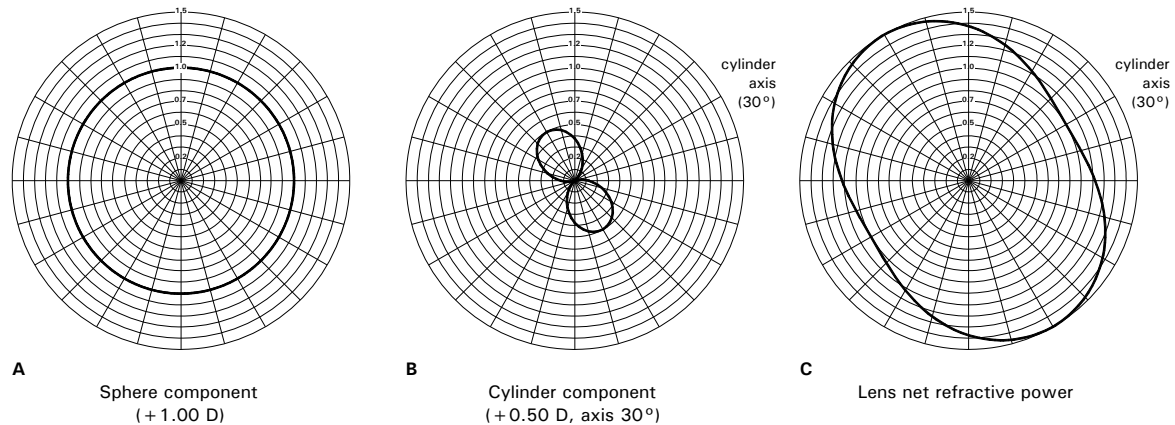
Before we continue, let me mention the small wrinkle about stating the angle of the cylinder axis. As stated in ordinary scientific work, the angle of the cylinder axis can only vary over the range 0° through

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<sup>3</sup> We must, however, be careful when considering the power of a cylinder lens at oblique angles. The refractive effect at such angles is not a simple as might be suggested by the power attributed to that angle.

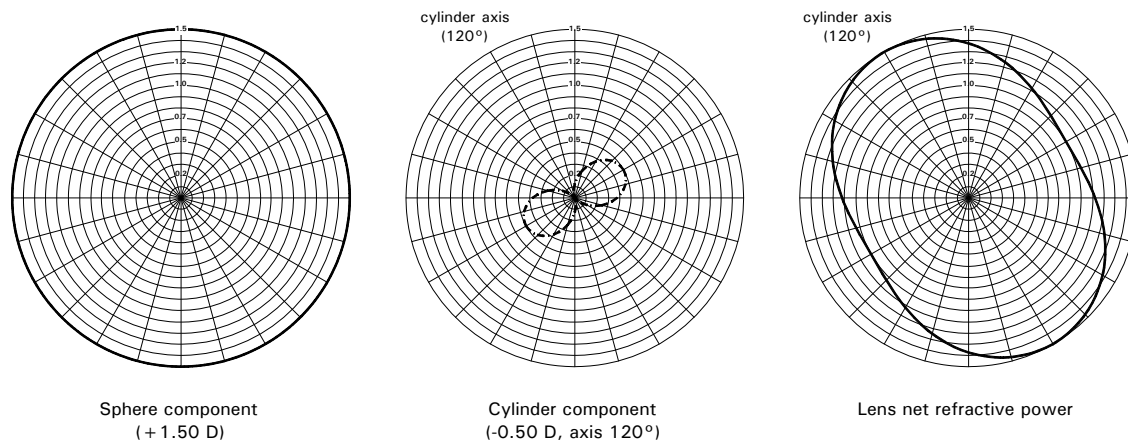
infinitesimally less than  $180^\circ$ . In optometric practice, the range of the angle is considered to be from infinitesimally greater than  $0^\circ$  through  $180^\circ$  (that is, we never write " $0^\circ$ ").

In figure 3, we see one example of the combination of a spherical and a cylindrical component:



**Figure 3. Composite lens-plus cylinder-power plot**

In figure 4 we see a different example:



**Figure 4. Composite lens-minus cylinder-power plot**

Note that the result here is identical to the previous case.<sup>4</sup> This is reminiscent of the ways we might make an ellipse. We might start with a circle of small diameter, and stretch it in the direction of the

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<sup>4</sup> The specific mathematical variation of the power of a cylindrical lens with angle makes this equivalence exact.

ellipse's major axis. Or we might start with a circle of large diameter, and shrink it in the direction of the ellipse's minor axis.<sup>5</sup>

We can of course make a single lens that will exhibit this overall behavior. A simple version (although not often used in modern ophthalmic practice) would have a front surface that is a portion of a sphere (implementing the "spherical" component of the overall power) and a rear surface that is a portion of a cylinder (implementing the "cylindrical" component of the overall power). This might be done with the cylinder surface either convex or concave, following the composition intimated by figures 3 or 4, respectively. The behaviors of the two will be identical (if we put aside some small wrinkles).

## 5 LENSES FOR VISION CORRECTION

### 5.1 The basic concept

In the application of eyeglass lenses, a role is played by both "spherical" (rotationally-symmetric) and "cylindrical" refractive behavior.

To correct for hyperopia (farsightedness), we provide a converging lens (net convex, with a plus power) to shift the range of focusing ability "closer". (Photographers do the very same thing with an *auxiliary closeup lens* to allow their cameras to focus at a closer distance than they would otherwise be able to.)

To correct for myopia (nearsightedness), we provide a diverging lens (net concave, with a minus power) to shift the range of focusing ability "farther".

In astigmatism, the eye's lens has a different refracting power in different directions, the maximum in a certain direction and the minimum in the direction at right angles to that. (The specific direction varies from person to person, from eye to eye.)

We could compensate for that by using a cylindrical lens with a plus power equal to the difference between the eye lens' maximum and minimum power (in a given state of focus), with its cylinder axis aligned with the eye lens' direction of maximum power (so the power of the cylindrical lens adds to the lesser-than-ideal power of the eye's lens).

Or we would use a cylindrical lens with a minus power equal to the difference between the eye's maximum and minimum power (in a

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<sup>5</sup> Notwithstanding this metaphor, the plot of the power of such a composite lens (or in fact of a cylinder lens by itself) is not an ellipse.



given state of focus), with its cylinder axis aligned with the eye lens' direction of minimum power (so the power of the cylindrical lens subtracts from the greater-than-ideal power of the eye's lens).<sup>6</sup>

Now, for a person having, for example, both hyperopia and astigmatism, we can visualize a sandwich of two lenses, a spherical lens to shift the focusing range (to overcome the hyperopia), as was just discussed, and a cylindrical lens "trimming out" the difference in the eye lens' refractive power in different directions (to overcome the astigmatism).

Of course, as we discussed earlier in a more abstract context (figures 3 and 4), we can make a single lens that does the same thing as that sandwich. A simplistic view would be a lens with a spherical surface on one face and a cylindrical surface on the other—two different ways.

Or we could visualize a lens with one face planar ("plano") and the other face having a compound curve, with one curvature along a certain direction and a different curvature along the direction at right angles to that.

In fact such a compound curve is found at the surface of a recognized three-dimensional figure, the *torus*. (A doughnut is nominally a torus in shape, as is a "rootbeer barrel".)

As a result, a lens having both spherical and cylindrical aspects to its refractive power, especially when conferred on one surface, is often referred to as a *toric* lens.

## 5.2 Interaction between spherical and cylindrical components

Assume that astigmatism is present, and suppose that the spherical lens we spoke of as correcting the person's range of accommodation was the best compromise for focus in the two critical directions—the one where the eye's refractive power was greatest, and the one where it was least (said to be the two *meridians* of the eye).

Now, if we chose to add a cylinder lens component with a plus power along the meridian in which the eye had the least power, the magnitude of its power being the difference between the eye's power along that meridian and its power along the other meridian, the overall "optical system" (eye plus corrective lens) would now have equal power along both meridians. Thus the astigmatism would have been

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<sup>6</sup> Note that the starting spherical power would have to be different in these two cases.

overcome. But that power would not be ideal in either direction—it would be uniformly too high in both directions.

To get the ideal result, we would also need to decrease the spherical lens power by half the magnitude of the cylindrical component.

If, on the other hand, we chose to add a cylinder lens component with a minus power along the meridian in which the eye had the most power, the magnitude of its power being the difference between the eye's power along that meridian and its power along the other meridian, the overall "optical system" (eye plus corrective lens) would now have equal power along both meridians. Thus the astigmatism would have been overcome. But that power would not be ideal in either direction—it would be too low in both directions.

To get the ideal result, we would also need to increase the spherical lens power by half the magnitude of the cylindrical component.

In summary, if we had determined the ideal spherical correction as a compromise for focus along the two meridians, then when we introduce a cylindrical component to overcome astigmatism, we must change the spherical power by the negative of half the cylindrical power (observing the algebraic signs).

## **6 THE PRESCRIPTION**

### **6.1 General**

An eyeglass prescription is a specification for the lenses in the glasses. It is done in terms of the model we saw above, in which the overall refractive pattern of the lens is described in terms of the joint effect of two hypothetical lenses, one spherical and one (only present if there is a correction for astigmatism) cylindrical.

Recall that, as we saw in figures 3 and 4, the identical lens behavior could be conceptually implemented with either of two "recipes"; for that particular example, we could combine:

- A spherical lens with power  $+1.00$  D
- A cylindrical lens with power  $+0.50$  D and axis  $30^\circ$

or

- A spherical lens with power  $+1.50$  D
- A cylindrical lens with power  $-0.50$  D and axis  $120^\circ$

And either of those two "recipes" could be viewed as specifications of the identical lens behavior (which would then apply to lenses made with either recipe).

## 6.2 Two systems of notation

Either model we saw above could be used (at our choice) as the premise for specifying a certain lens **behavior** in a prescription. It turns out that when the prescription is written by an ophthalmologist (a physician and surgeon specializing in the eyes), it would be in the first form (the cylinder component always having a plus power), called the “plus cylinder” form.

When the prescription is written by an optometrist (a Doctor of Optometry, qualified and certified to examine eyes and issue eyeglass prescriptions), it would be in the second form, (the cylinder component always having a minus power), called the “minus cylinder” form.

This is not only the result of accidental historical “diversity”. At one time, licensed optometrists were, in many states, not allowed to prescribe nor administer any medication, which in most cases included the “drops” that could be used by ophthalmologists to disable the eye’s accommodation mechanism so that its attempt to keep the image on the retina in focus would not disrupt the refraction process.

At an earlier time, the prescription was thought of not as just an optical specification for the corrective lens but actually a recipe for the lens construction.

Also at that time., it was most common, in making corrective lenses to have the cylinder component executed on the front of the lens, and “mechanical” considerations suggested that this be done with a plus cylinder component.

Again thinking of the prescription as a “recipe” for making the lens, an ophthalmologist would prefer to write the prescription in plus cylinder form. And that form would most directly come from a refraction using plus cylinder lenses.

And the most convenient strategy for doing a refraction called for the eye’s accommodation mechanism to be disabled. And there were medications that would do that. In that *modus operandi*, it was essentially equally convenient to refract for the cylindrical component using plus or minus cylinder lenses in the refractor. So this all worked out well for an ophthalmologist.

But not so well for an optometrist, who most likely could not use such medication.

That being the case, the best strategy for avoiding attempts of the eye’s accommodation mechanism to change the focus of the eye while it was being measured worked most conveniently using minus

cylinder trial lenses.<sup>7</sup> And the prescription form that flowed most directly from that used minus cylinder notation.

But what about when the lens was to be made, most conveniently with a plus cylinder component? Well, of course lens makers were well equipped to change the form of the “specification” from a minus cylinder basis (on a prescription written by an optometrist) to a plus cylinder basis (needed to set up the equipment to grind the front of the lens).

Now, today, almost all of those fascinating situations have moved into folklore. In most states, optometrists can (if they wished) administer medication to disable the eye’s accommodation mechanism while the eye was being “refracted”. And today, it is most common to execute the cylinder component on the back surface of the lens, where the same “mechanical” considerations alluded to earlier suggest the use of a minus cylinder component.

But the die was cast. Ophthalmologists have refractors equipped with (only) plus cylinder lenses in their lens wheels, and trial lens sets that include (only) plus cylinder lenses; optometrists have refractors equipped with (only) minus cylinder lenses in their lens wheels, and trial lens sets that include (only) minus cylinder lenses.

### 6.3 Format of the prescription

In a prescription for eyeglasses, there is a line (or section) for each eye. In each, there are three parameters stated:

- The power of the spherical component (could be zero, often written as “plano” rather than “0”). This is normally in increments of 0.25 D.

If there is a cylindrical component:

- Its power (normally in increments of 0.25 D)
- The angle of its cylinder axis (normally in increments of 5°, from 5° through 180°).

Often, the indicators OD (from the Latin, *oculus dexter*) and OS (*oculus sinister*) are used for the right and left eyes, respectively. (OU—*oculus uterque*—indicates both eyes.)

We will give our first complete example in the “plus cylinder” system.

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<sup>7</sup> This is discussed in detail in the companion article, “Plus and minus cylinder notation in ophthalmology and optometry”, probably available where you got this.

There a complete prescription might look like this:

OD +1.25+0.50 X 130

OS +1.50+0.75 X 25

Sometimes the decimal points are omitted (and all powers stated to two decimal places), so it would look like this:

OD +125+050 X 130

OS +150+075 X 25

There are many other variations in style.

For the very same pair of lenses, under the “minus cylinder” system, the prescription might look like this:

OD +1.75-0.50 X 40

OS +2.2-0.75 X 115

#### 6.4 Conversion between equivalent forms

If we have a prescription in either “plus cylinder” or “minus cylinder” form, we can easily convert to the other form, this way:

- Add the present sphere and cylinder powers (observing the signs). The result is the new sphere power.
- Reverse the sign of the cylinder power; the result is the new cylinder power.
- Add or subtract  $90^\circ$  to/from the current cylinder axis angle (so that the result will be non-zero and plus but not over  $180^\circ$ ). The result is the new cylinder axis angle.

#### 6.5 Variations

Sometimes the prescription will be written this way (for one eye only shown):

OD +1.25 DS+0.50 DC X 130

where “DS” means “diopters, sphere” and “DC” means “diopters, cylinder.

Sometimes when the needed sphere power is zero, the prescriber will write “plano” (rather than 0.00). That is because a lens with both surfaces flat (“plano”) will have a power of zero (like a window), so “Plano” becomes a metaphor for a zero-power lens.

## 7 BIFOCALS

### 7.1 The basics

Especially when presbyopia is present, there may be no single corrective lens prescription that will allow the eye to focus on both near and far objects. We address this with *bifocal glasses*, which have two areas of different refractive power, one (covering most of the lens) used to see distant objects and the other (called the near-vision segment, or just the “segment”), usually at the bottom of the overall lens, used to see near objects.

Normally, the properties of the segment are the same as for the base lens except that the spherical power is greater. This is specified in the prescription in an incremental way: an “add” value that tells how much greater is the sphere power in the segment than in the rest of the lens for that eye:

OD +1.25 +0.50 X 120 add 1.75  
 OS +1.50 +0.75 X 22 add 1.75

Very commonly the add value is the same for both eyes (often being determined empirically only by the distance at which the person wants best “near-vision” results). In that case, the add value may be stated only once in the prescription:

OD +1.25 +0.50 X 120  
 OS +1.50 +0.75 X 25  
 add +1.75

In either case, the implication of this is that the actual definition of the **near vision segment** for the right eye (OD) is:

+3.00 +0.50 X 120

That is, the sphere power in the segment (+3.00) is the sum of the base sphere power (+1.25) and the “add” sphere power (+1.75).

### 7.2 Near vision effectivity error (NVEE).

As mentioned above, In other parts of the optics field, we generally describe the refractive property of a lens in terms of its *focal length* (that is, the *effective focal length*, to be precise). We also speak of the inverse of that, the *refractive power* (which I sometimes here will call the *effective refractive power*, to avoid any ambiguity.)

Then, to determine, for an object at a certain distance, at what distance will the image of that object be created, all we need to know is the (effective) focal length.<sup>8</sup>

It does not matter how the lens is implemented: it might be thin, or it might be thick, or it might be a compound of two or more lens elements.

Again as mentioned earlier, In the field of vision correction lenses, we characterize the refractive property of a lens in terms of its (back) *vertex power*. It is defined as the reciprocal of the *back focal distance*.

For an object at infinity (or, in realistic terms, a great distance), any lens having a certain (back) vertex power, regardless of the details of its construction. will behave the same as to its effect on vision.

However, with respect to objects at a more modest distance (with which we are concerned in the entire area of "near vision") not all lenses with a certain (back) vertex power will have the same effect on vision.

But we generally treat them as if so. For example, we may consider the stack of lenses in a trial frame that make up the "model" of the ideal near vision corrective lens to be, by virtue of the sum of the power of its "main" and "near vision add" sphere lenses, the same as an actual corrective lens made to have that same sphere power in its "near vision segment" region.

But in fact the "composition" of the testing lens stack is almost certainly different from the composition of the actual corrective lens made from that prescription. Thus, even though the two "lenses" may have identical back vertex powers, their effect on near vision may not be the same.

This discrepancy is quantified by the property *near vision effectivity error* (NVEE). How is that defined?

First consider the distance at which the test lens setup (or the actual corrective lens) brings to a focus the rays originating from point on a target at the near vision distance of interest. We take the reciprocal of that distance (as if it were a focal length and we were determining the

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<sup>8</sup> That's not exactly so. Both the object and image distances are defined as being from the 1st and 2nd principal points of the lens, and so for precise calculations, especially when the object and image distance are fairly small compared to the focal length, we must know where those two points are located.

corresponding power), and denominate it in diopters. This is said to be the *vergence* of the rays leaving the lens toward the eye.

Then we consider the distance at which a fictional thin lens having the same vertex power as the lens of interest (in a thin lens, the *vertex power* is the same as its *effective power*) would bring to a focus the rays originating from a target at the near vision distance of interest. Again we take its reciprocal, getting a second value in diopters. This is said to be the *vergence* of the rays that would leave this fictional lens toward the eye.

The first vergence value minus the second vergence value is the *near vision effectivity error* of the lens in question.

It is worth noting that the NVEE usually only becomes of consequence for thicker lenses, which typically means lenses of a large plus power.

How this is taken into account in the design of a corrective lens is beyond my current ken, and thus beyond the scope of this article.

## 8 PRISM CORRECTION

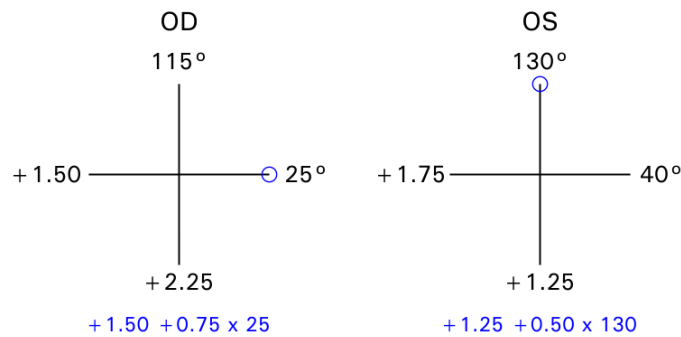
In some cases, the “pointing directions” of the two eyes are not consistent (at least not happily). This can cause a problem with the brain “fusing” the images from the two eyes (in some cases causing “double vision”).

This can be mitigated by including in one or both lenses a “prism” component. Its purpose is just to deflect the line of sight, in a certain direction, by a certain angle. Further discussion of this aspect of eyeglass optics and the prescription notation for it is beyond the scope of this article.

## 9 THE OPTICAL CROSS

We almost never see polar plots of refractive power, such as we see in this article, in the literature of practical eyeglass optics. But there is another useful graphic convention that is often used: the *optical cross*. Its purpose is to show the overall result of the spherical and cylindrical components: what the finished lens does in terms of refractive power. It is in fact sometimes used to define the desired properties of a lens (rather than using the “prescription” form).





**Figure 5. Optical crosses—based on plus cylinder prescription**

We see it in Figure 5, displaying the characteristics of these two arbitrarily-defined lenses (expressed here in “plus cylinder” form):

OD + 1.50 + 0.75 X 25

OS + 1.25 + 0.50 X 130

The two legs of the cross are said to represent the two “principal meridians” of the lens: the directions of maximum and minimum net refractive power of the lens. One is always the cylinder axis; the other is always at right angles to that (the direction in which the cylinder lens has its maximum power, of whatever sign), but not necessarily respectively. (The cylinder axis meridian will have the minimum power under the minus cylinder form, and the maximum power under the minus cylinder form.)

Each leg is labeled at its upper (or rightmost) end with the angle of the meridian it represents, and at its lower (or leftmost) end with the net power in that direction. Note that the cross is not drawn with these meridians at their actual angles.

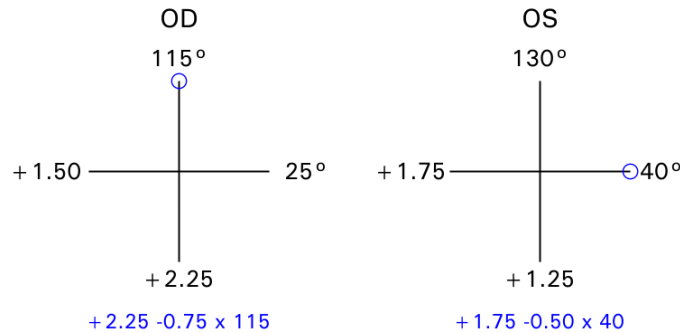
For the benefit of the reader, I have stated the prescription for each lens under its cross representation. This would not usually be the case in practice; the cross is itself an alternate way to describe the lens’ desired performance. Also for the convenience of the reader, I have indicated the meridian that corresponds to the cylinder axis with a small circle around its tip. Again, this would not be seen in the actual use of the optical cross. (These two “private” annotations are in blue, for the benefit of readers seeing this in color.)

We note that on the cylinder axis, along which the cylinder component has no power, the net power is just the sphere component. On the other meridian, where the cylinder component exhibits its maximum (and stated) power, we note that the net power is the algebraic sum of the sphere and cylinder powers.

Now we know that this pair of prescriptions, in “minus cylinder” form, defines exactly the same pair of lenses:

OD + 2.25 -0.75 X 115

OS + 1.75 -0.50 X 40



**Figure 6. Optical crosses—based on minus cylinder prescription**

In figure 6, we show the optical crosses for that outlook:

Notice that these are identical to the ones in figure 5 (except of course for my “private” annotations, which would not appear on a real optical cross). (Seems reasonable—they are for the same lens.)

So, seeing an optical cross in normal form, how can we tell which of the two meridians corresponds to the cylinder axis? Well, we can’t, and we don’t need to in order to understand what lens behavior the optical cross defines.

Recall that the distinction between one or the other meridian corresponding to the cylinder axis is an artifact of the prescription scheme, with its alternative plus cylinder and minus cylinder forms. The optical cross shows the final result in terms of lens behavior, not in terms of one of two alternative “recipes” for defining it.

From an optical cross, can we determine the “regular form” prescription for the lens it defines? Yes, but we must first choose whether we want it in the plus or minus cylinder form. The math is simple, but I won’t show it here.

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